Possibilities of recovery from kinetic energy of a vehicle <u>Update of November 7th, 2009</u>

Elements of pre-sizing of some devices for recovery on classic cars



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Note:

This document doesn't have the pretention to be exhaustive and only provides typical sizes

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Generalities and purposes

The recovery of kinetic energy from a vehicle is not a new idea although it is few spread: most vehicles slow down while converting the totality of their kinetic energy into heat in their brakes. This energy is freed in the atmosphere and it is definitely lost for the vehicle. In fact, one can separate the sources of energy in 2 main categories:

- The irretrievable energies: these are energies which can't only be unstored inside the vehicle while it is running. Either they are not with reversible working, either devices to regenerate these energies don't exist in the vehicle. Examples: the chemical energy of a fuel that it is impossible to reform after its combustion, the electric energy of batteries not equipped with a regenerator.
- The retrievable energies: they are energies which can be alternately stored or unstored *inside* vehicle *while it is running*, under the condition that it gets adequat devices: then they become with *reversible working*. Subsequently, the unstorage of this energies allows to make advance the vehicle. Inversely, one can stock these energies merely while making roll the vehicle and without needing for sources of energy outside the vehicle (ex: a station service, the electric network, the sun, wind.). Examples:
 - The mechanical energy: it can be stored notably in a vehicle with the help of springs (principle of the friction-cars), either with the help of wheels with high inertia, either under electromagnetic shape with the load of capacitors ("supercaps") or spools.
 - o The chemical energy: it is stored in accumulators (or batteries): the reactions of spontaneous oxydoreduction or not (electrolysis) permit to store and unstore this energy (the unstorage produces an electric energy).
 - o The air / hydraulic energy : a gas (or a fluid) under pressure constitutes the tank of energy (compressing the fluid: storage; expanding : unstorage, like a true "pneumatic spring").

The recovery of the kinetic energy from the vehicle is a major stake to decrease its consumption of irretrievable energy. Indeed, under the hypothesis where rubbings are weak (perfected tires or haulage on railroad tracks, tidy mechanisms to transmit the power toward the wheels and motors limiting the losses by rubbings), a vehicle could cruise thousands of km with an initial relatively weak stock of irretrievable energy: it would take the advantage of coming down or braking to store the recoverable energies, then this storage will be useful for it to face again the rises and to defeat the rubbings which will always be present, even though they are reduced.

The irretrievable energy of most present vehicles is a chemical energy from burning hydrocarbons with dioxygen, well known as "fuel or gasoline and Diesel". *The reduction of the consumption of fuels is a overwhelming evidence* both from an economic point of view (continuous raise of the price at increased speed after the peak oil), that from an ecological point of the view (the combustion of fuels releases carcinogenic gaseous pollutants and greenhouse effect mainly because of the CO2). Then one can wonder that about 2008, nearly all vehicles waste their fuel without trying to recover their kinetic energy (with the exception of some very rare hybrid or electric vehicles). In fact, it is only a consequence of the combination of 4 factors:

- The influence of the oil industry for that the consumption of oil of the developed countries and soon the one of big emergent areas (South America, China, India.) is a very important goal to absorb their investments to prospect of 'oily layers'.
- <u>The condescension of the States</u> that, while taxing fuels massively, insidiously exploit the situation in spite of declarations of circumstances on the protection of the environment and the importance of their energetic independence.



- <u>The permanence of the drivers</u> that in their majority, seem to resign themselves in the gas-station, and especially don't require any efficient products, but comfortable and powerful.
- The under investment of the cars' manufacturers which, except rare exceptions, think of giving a valorizing picture of themselves while confiding to their R&D service the development of ecological prototypes, presented with a certain self-congratulation in the lounges of the car. The sincerity of the R&D services and the quality of their realizations contrast with the strategies of the manufacturers that, since the beginning, don't consider as necessary the introduction of these technologies in cars before 4, 5, 6 years. That is to say when the market or a legal setting the will force them to do it, what risks to be a *very* far future, unless the barrel of oil reaches some peaks earlier than foreseen, following an overestimate of the exploitable "reserves".

Yet, the automotive manufacturers pretend to be constantly innovating. It is both true and false. Certainly, as regards to the geometrical design, the cars constantly evolve, but the boundary between a novelty and an innovation is sometimes fuzzy in this domain. On the technical plan, that is the one where SYCOMOREEN stands more gladly, it is necessary to bring for their credit numerous innovations, in particular in the domain of the passive or active security, of the frame, in the transmission of the power to the wheels, as well as in the electronic management of the combustion. But in the same time, it is necessary to notice that since the beginnings of the car, either since 110 years, the architecture of the thermal engines remained frozen: some cylinders contain the pistons of which the motion of coming and going is converted via a rod into a rotary motion by a brace. We won't come back here on the necessary change of this architecture, but let merely say that it is cumbersome and that consequently, it forbids the presence of believable devices of storage of energy in a vehicle. On the contrary the emergence of very compact engines with same power as the current motors would make possible a meaningful storage of retrievable energies thanks to the gains of space that they would offer in the vehicle. With the Machines with Rotary Pistons and Controlled Beating (MPRBC) and the Octagonal Piston with Controlled Deformable Geometry (POGDC), SYCOMOREEN proposes very meaningful breakthoughs in the design of such engines and wish to widen slightly here the field of its works with the following objectives:

- To describe qualitatively the possible conversions of energy in a vehicle
- To value the size of devices to store the kinetic energy of a 1000 kg vehicle running at 72 km/h in order to see what is foreseeable or not on a car.
- To sketch technological solutions allowing the energetic exchanges within the vehicle.

I.1. The chemical energies

I.1.a) The hydrocarbons chemical energies (except coal)

They are numerous, and most of them are currently widely used. One can note:

- o The gasolines: supercarburant, plombless... for engines with spark ignition.
- o The gasoil and heavy oils used for engines with spontaneous ignition (Diesel).
- o The liquefied gas of petroleum (LPG) used as a complementary fuel for the gasoline motors. Their introduction in France at the end of the nineties proved to be a failure since the rate of equipment of the vehicles gas is lower to 5%, although their release of pollutants are reduced and that the State, in his big goodness, limits the taxes on this type of fuel, yet eminently fossil and polluting enough (NOx,CO₂).
- The *fuels produced from food organic matters*: methanol, esters... which are starting to be significant, notably in Brésil, but directly compete with the human food matters and are not as ecological as one imagines it. The *major ecological argument* of the green fuels is an *almost-deletion of the carbon releases* (CO2) in the atmosphere: the photosynthesis of the plants uses big quantities of CO2 and reject the dioxygen (O2). This is how the photosynthesis of the plants used to get biofuel will roughly consume the quantity of CO2 that the combustion of biofuel will produce in advance, but what are the CO2 releases of production and transportation of biofuels? What impact have the pesticides necessary to get the required outputs? Is it necessary to starve the planet *to feed*... the most badly designed cars because they aren't saving anything?
- The *fuels from biomass*, have the advantage to use organic parts which are usually not or few valorized, by various process (biomethanisation, thermolysis, gazeification...); they are a way of research which has to be surely worked, but which will not allow to sustain the present waste of hydrocarbons.

These energies from hydrocarbons are *extremely energizing* (heat produced by combustion of about 20 000 kJ/kg of fuel) and easy to manipulate under the liquid shape, *but they are pollutant*.

The conversion of this chemical energy is *irreversible*: it means that *one can only unstore this energy* inside the terrestrial vehicle. One schematically exploits the chemical energy of the hydrocarbons so:

$$\underbrace{C_x H_y O_z N_t S_w}_{\text{chemical energy}} \xrightarrow{\text{irreversible combustion}} \underbrace{X CO_2 + \frac{y - 2v}{2} H_2 O}_{\text{supposed entire with air}(4N_2, O_2)} \xrightarrow{\text{supposed entire with air}(4N_2, O_2)} \underbrace{t \ NO_2 + u \ N_2 + v \ H_2}_{\text{expanding}} \xrightarrow{\text{and thermal losses}} \text{mechanical energy}}_{\text{order the minical energy}} \times \underbrace{WSO_2 ...}_{\text{Very hot gas under pressure}}$$

I.1.b) The chemical energies without carbonated content

They are used under electric accumulators, of which the most spead is the plomb-battery for cars (electrolytic accumulators). They are working on *reactions of oxydoreduction* to create an electric current. *Their big asset is the reversibility of these reactions*; indeed by applying a reverse voltage, *the accumulator gets energy back*. However, the number of cycles storage/unstorage is limited, but quite high (around 2000, it depends on the used technology of battery). The increase of the number of cycles storage/unstorage of accumulators is the matter of a lot of research, because the applications are numerous (batteries of laptops, cars, aerospatial devices). One schematically has:

 $\underbrace{reactants}_{chemical\ energy} \xleftarrow{spontaneous\ oxydoreduction}_{consuming\ the\ reactants} \underbrace{voltage/current}_{electric\ energy} \xleftarrow{electric\ motor}_{electric\ generator} \underbrace{mechanical\ energy}_{mechanical\ energy}$

One notices the double arrows: it means that *one can store and unstore this chemical energy* inside the vehicle, and that with 2 ways: either the thermal engine runs a generator which loads the batteries, either the kinetic energy of the car is turned into electric energy while the vehicle is slowing down. Thus *a car will brake without actuate the mechanical brakes*: the energetic saving is very important as *a big part of the kinetic energy*, usually lost entirely by Joule effect in the brakes *is recovered here* (fig. I.). This energy will subsequently be useful to launch again the vehicle, and so on ...

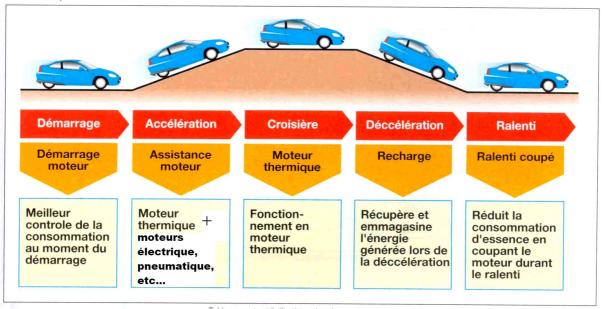


Fig. I..: strategies to recover the kinetic energy and to sustain the thermal engine of an hybrid vehicle.

Let's signal a variant in the storage of electricity as the use to the supercaps well wished as energizing tampons because of their short time of response and their compatibility with strong amperages.

Finally, one can mention like oxydoreduction in gaseous phase the particular case of the fuel cells dihydrogen / dioxygen which knew a renewal of interest lately, after having endured the clutter of the tanks and various circuits a long time. However, the big scale industrialization of such devices is not considered before several years and dihydrogen is more an energetic problem (because of its steps of production, storage, transportation and unstorage) than a real solution.

I.2. The pneumatic/hydraulic energies

They are energies with a quite similar behaviour and *work reversibly*, like the chemical energies of oxydoreduction:

They can also be used to brake or launch the vehicle while limiting the Joule losses in the brakes.

I.3 The mechanical energies

They are the wheels of inertia previously thrown at high speed, which, linked with a transmission, assure the traction of the vehicle. In the phases of acceleration, the wheel slows down, in the phases of deceleration, the wheels is increasing its rotary velocity (kinetic energy recovery). These devices require very complex transmissions to adapt the speed of the storing wheels to the one of the rolling wheel (epicycloidal gearings and/or continuous variation transmission, with clutches and an electronic management). Besides, the wheels are generally heavy, cumbersome, or even dangerous in the vehicle and always need an engine to relaunch them if necessary. In spite of these difficulties, some studies try to introduce this technology in Formula 1 (KERS project: Kinetic Energy Recovery System). The other alternative of storage is to impose strength in a springs (traction / compression or torsion); then the energy is stored as elastic potential energy. The relaxation of the spring *frees* this energy back whereas making it under tension *stores* the kinetic energy while slowing or coming down.

I.4. The renewable energies

They are solar and wind energies. They can't currently be used to move the car, except if it is ultra light (less than 300kg with passengers), what it is usually not the case. They are not reversibles: the vehicle can't do back the wind which has pull it or send back the light it has got. Nevertheless, the solar energy via photovoltaic cells can run the small electric accessories or load the batteries, with running car or not. For example, a solar car has moved across the Australia in the South/North direction. Unfortunately, the solar panels can't be very big on a car (surfaces limited at the roof, at the engine's hood and to the wings), and they are expensive and fragile.

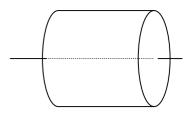
II. Elements of pre-sizing of the devices for storage

II.1. Previous hypotheses

The storage of energy is justified for all terrestrial vehicles, especially for the cars which move in city. The ideal would be that with a quantity of initial energy, the car can propel itself until the maximal speed in urban situation, then by recovery while braking, accelerate until roughly the same speed *without additional energy*. The urban speeds are located between 0 km/h (red lights or plugs) and 70 km/h (peripheral). While rounding the maximal speed to 72 km/h, either 20 m/s, the energy to recover during

a complete braking is for a rolling mass of m=1000 kg: $E_c = \frac{1}{2}mv^2 = 200\ 000\ J$. The storage must be

efficient (at least 90% of the recovered kinetic energy), fast (duration of the braking time, at most some score of seconds) and must allow the vehicle to brake without actuating the mechanical brakes, except at the last moment to immobilize the vehicle completely. Otherwise, the devices of storage must have a reasonable size. One gives itself by hypothesis a volume dedicated to the storage of the energy of 20 000 cm3.



II.2. The mechanical storages

II.2.a) With wheels of inertia

One considers a cylinder of steel with a volumic mass of 7850 kg/m³, a radius R=25 cm and a height h=10 cm. Its volume V is worth : $V=\pi R^2h=19635$ cm³, and $m=\rho V=154.13$ kg

And the inertia momentum J regarding the axis of revolution is : $J = \frac{m_{cylinder}R^2}{2} = \frac{\rho\pi hR^4}{2}$

The rotary kinetic energy of this solid: $E_{rotation} = \frac{1}{2}J\omega^2 = \frac{\rho\pi hR^4}{4}\omega^2$ with ω its rotary velocity.

While supposing an angular velocity of 2800 RPM (like a thermal engine at a middle load), to launch this wheel of inertia from 0 to 2800 tr/min (293,2 rad/s) is storing the following energy:

$$E_{rotation} = \frac{1}{2} J \omega^2 = \frac{\rho \pi h R^4}{4} \omega^2 = 207058 \text{ J}$$

what allows theoritically to throw again the vehicle at about 70 km/h. But it is far to be perfect:

- The weight is important: only the inertia wheel weighs here more than 154 kg, and it has to be foresee magnetic transmissions (or mechanical CVT), which are heavy and cumbersome too.
- The mechanical complexity of this transmissions entails high costs.
- To accelerate quickly the inertia wheel (brutal braking), it will oppose a strong torque in $\sqrt{d\omega}$ ». This torque is likely to be too high and could block the rolling wheels: thus an

electronic management imposes itself (clutch or regulation of the rate $\omega_{wheels}/\omega_{inertia\ wheel}$).

II.2.b) With springs

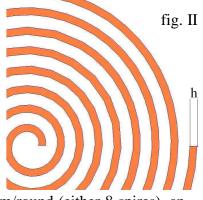
For the spiral springs in torsion, the rotary stiffness is given by:

 $K = \frac{Ebh^3}{12I}$ With: E the Young modulus of the material (steel, 210 GPa),

b the depth of the spring, h the thickness of the rolled section (fig. II) and L the developed length of the section.

In the approximation of the linear distortions, the back torque of the spring while fixing one of its extremities and rotating the other of an angle

$$\theta$$
 is $C_{back} = K\theta$. The stored elastic energy and is given by $E = \frac{1}{2}K\theta^2$



By choosing a maximal radius of 25 cm, a radial increase of 2,875 cm/round (either 8 spires), an initial middle radius of 2 cm, a thickness h of 12 mm and a depth b of 10 cm, one gets: L = 6,79 m, K = 445,25 N.m/rad, the spring's volume: 19635 cm³

And for a rotation of 2.5 rounds : $C_{back} = 6994 \ N.m$ and $E = 54.931 \ J$, like a re-launch until 38 km/h.

The aspect of the spring is shown on figure II. *The stored energy is interessant*. However one would have to check that deformations of the metal are still *reversibles*. The important back-torque imposes the use of reducing gearings with a rate of about 10 between the rotary motion of the wheels and the shaft bound to the center of the spiral. Moreover, one has to foresee a device to invert the motion (with clutch for example), otherwise, *the relaxation of the spring make the car go back*!

For the same volume, the spiral spring stores 50 000 J, either 4 times less than the inertia wheel. Nevertheless, it is a lot lighter and, *a priori*, it doesn't need continuously variable transmission (CVT).

For the springs in traction/compression, the elongating stiffness is worth : $k = \frac{Gd^4}{8nD^3}$

With: G the shearing modulus of the material (for steel: $G = \frac{E}{2(1+v)} \approx 80$ GPa with v=0,28), d the diameter

of the rolled up thread, n the number of spires for the springs, and D the average diameter of the helical spring. If we stay in the elastic range of the spring (reversible deformations),

the back-strentgh of the spring which has been elongated by x is $F_{back} = kx$

and the stored elastic energy is $E = \frac{1}{2}kx^2$.

The constraint inside the springs is given by $\sigma = \frac{8F_{back}D}{\pi A^3}$

If we choose a traction spring with joined spires and the following features: d = 34 mm, R = 85 mm, initial length 30 mm, elongation x = 30 mm, one gets the spring with à 9 spires as shown on figure III (in initial and elongated configurations) with the following characteristics:

- Stiffness of the spring : k = 308 267 N/m
- Strength required to elongate by x : F = k x = 92480 N
- Stored elastic energy: 13 872 J
- Mechanical constraint: 1 018 MPa
- Volume of the *elongated* spring: 19611 cm³



fig. III

Here is the stored energy clearly less than with the spiral spring. It is because of the loss of space at the center of the spring. The constraint is high, but the steels HLE have elastic limits until 1500 MPa. Finally, the required strength for elongation is important: it is like the pressure of 100 Bar on a 100 cm² surface.

II.3. The pneumatic/hydraulic storages

II.3.a) Case of a pneumatic energy

Choice of the fluid:

It will be a gaseous fluid: it will grant the energetic exchanges, and the cheapest and the most available is the air of the atmosphere.

- Advantages:
 - The pneumatic compressors or motors are quite cheap.
 - A tank of air under pressure is less dangerous than a tank of hydrocarbons or explosive gas, especially if the car is accidentally burning.
 - The compressor could be *integrated* inside the engine :
 - One can really think up to convert one of the rooms of combustion into an intermittent compressor with the help of variable valve timing(VVT) and another valve routing the gases alternately towards the exhaust or the pressurized tank. One can even give up definitely using this room like a combustion chamber and to manage it exclusively as a pneumatic compressor/motor (the bi-directional valve becoming then useless).
 - Thus one can reach pressures of at least 50 Bars, especially if the rate of compression of the room is big and the dead volumes are small.



Drawbacks

- The output of pneumatic motors is often less than the one of hydraulic or electric motors; thus the energetic balance may be weaker.
- It seems that it is not usual to use reversely a pneumatic motor, as a compressor. But a reversible pneumatic device would be more compact than 2 dedicated devices.

One will value the mechanical work W that has to be given to a gas to compress it from a step $[P_1,V_1]$ to another step $[P_2,V_2]$ with $P_2>P_1$ and $V_1>V_2$. One will suppose a polytropic evolution like $PV^k=cte$ with k a positive parameter. The case where $k=\gamma$, with γ the Mayer's coefficient, is linked with an adiabatic reversible compression, which is also an isentropic change. For the perfect diatomic gases, $\gamma=1,4$. The case where k=1 corresponds with an isothermal compression because in this case, with the law of the perfect gases, $PV=nRT=cte\Rightarrow T=cte'$. One will use also $\tau=V_1/V_2$ the rate of compression.

The polytropic law give us while compressing:
$$PV^k = cte = P_1V_1^k \Rightarrow P = P_1\left(\frac{V_1}{V}\right)^k$$

$$W = \int_{V_1}^{V_2} -P dV = -\int_{V_1}^{V_2} P_1 \left(\frac{V_1}{V}\right)^k dV = -P_1 V_1^k \int_{V_1}^{V_2} \frac{dV}{V^k} = \frac{P_1 V_1^k}{1-k} \left[V_1^{1-k} - V_2^{1-k}\right] = \frac{P_1 V_1^k}{1-k} V_1^{1-k} \left[1 - \left(\frac{V_2}{V_1}\right)^{1-k}\right]$$

Also
$$W = \frac{P_1 V_1}{k-1} \left[\left(\frac{V_2}{V_1} \right)^{1-k} - 1 \right] = \frac{P_1 V_1}{k-1} \left[\left(\frac{V_1}{V_2} \right)^{k-1} - 1 \right]$$
 and finally $W = \frac{P_1 V_1}{k-1} \left[\tau^{k-1} - 1 \right]$

N.A.:
$$\tau = 16$$
, $P_1 = 1$ $Bar = 10^5 Pa$, $V_1 = 0.02$ $m^3 = 20000$ cm^3 , $W = 10 157$ J

If one wants 200 000 J (20 times more), it needs a tank of 400 L with air under pressure, what *it is very cumbersome to the scale of a car*, although this tank may be light *a priori*. Qualitatively, one has features *opposed* to those of the wheel with inertia.

At the end of this polytropic compression, the final température of the air seen as a perfect diatomic gas

with
$$PV=nRT$$
 will respect $T_1V_1^{k-1}=T_2V_2^{k-1} \Rightarrow \left[T_2=T_1\left[\frac{V_1}{V_2}\right]^{k-1}\right]$

If
$$k = \gamma = 1.4$$
, $\tau = 16$ and $T_1 = 20^{\circ}C = 293$ K, $T_2 = 888$ K either 615°C & $P_2 = P_1 \left(\frac{V_1}{V_2}\right)^k = P_1 \tau^k = 48.5$ Bar

If this pressurized air is stored for a very short time, it will not become cold and its relaxation will send back nearly the initial stored kinetic energy of the vehicle. On the contrary, if the expansion is not immediate, the air starts to cool until it reaches the ambient temperature T_1 with a constant volume V_2 . According to the conservation of the matter in the pressurized tank, the new available pressure P_2^{cool} will be then:

$$\frac{P_{2}^{cool}V_{2} = nRT_{1}}{P_{2}V_{2} = nRT_{2}} \Rightarrow \frac{P_{2}^{cool}V_{2}}{RT_{1}} = \frac{P_{2}V_{2}}{RT_{2}} \Rightarrow P_{2}^{cool} = P_{2}\frac{T_{1}}{T_{2}} = P_{2}\left[\frac{V_{1}}{V_{2}}\right]^{1-k} = P_{2}\tau^{1-k} = P_{1}\tau^{k}\tau^{1-k}$$
Thus
$$P_{2}^{cool} = \tau P_{1} \text{ N.A.} : P_{2}^{cool} = 16 \text{ Bar}$$

Now we're searching the mechanical work W that can be recovered while relaxing polytropically until the final step $\{P_{\text{final}}=1 \text{ Bar}, V_{\text{final}}\}$. The volume where P=1 Bar again will respect :

$$P_{2}^{cool}V_{2}^{k} = P_{final}V_{final}^{k} \stackrel{P_{final}=P_{1}}{\Rightarrow} V_{final} = \left(\frac{P_{2}^{cool}}{P_{1}}\right)^{1/k} V_{2} = \tau^{1/k} \frac{V_{1}}{\tau} \stackrel{\longrightarrow}{\Rightarrow} V_{final} = V_{1}\tau^{\frac{1-k}{k}} \text{N.A. } V_{final} = 9,06.10^{-3} \, \text{m}^{3} \simeq 9L$$

The polytropic law imposes while expanding: $PV^k = cte = P_2^{cool}V_2^k \Rightarrow P = P_2^{cool}\left(\frac{V_2}{V}\right)^k$

$$W = \int_{V_{2}}^{V_{final}} -PdV = -\int_{V_{2}}^{V_{final}} P_{2}^{cool} \left(\frac{V_{2}}{V}\right)^{k} dV = P_{2}^{cool} V_{2}^{k} \int_{V_{final}}^{V_{2}} \frac{dV}{V^{k}} = \frac{P_{2}^{cool} V_{2}^{k}}{1-k} \left[V_{2}^{1-k} - V_{final}^{1-k}\right] = \frac{P_{2}^{cool} V_{2}^{k}}{1-k} V_{2}^{1-k} \left[1 - \left(\frac{V_{final}}{V_{2}}\right)^{1-k}\right]$$

$$W = \frac{P_{2}^{cool} V_{2}}{1-k} \left[1 - \left(\frac{P_{2}^{cool}}{P_{1}}\right)^{\frac{1-k}{k}}\right] = \frac{1}{1-k} (\tau P_{1}) \frac{V_{1}}{\tau} \left[1 - \left(\frac{\tau P_{1}}{P_{1}}\right)^{\frac{1-k}{k}}\right]$$
Either
$$W = \frac{P_{1}^{cool} V_{2}}{k-1} \left[\tau^{\frac{1-k}{k}} - 1\right]$$

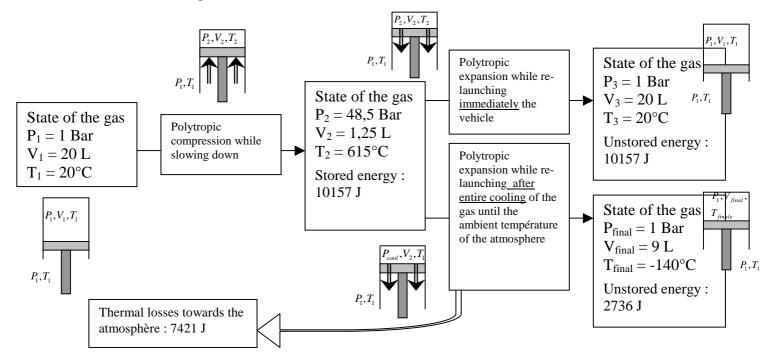
N.A: W = -2735,7 J (negative energy because it is given by the gas to the wheels of the car).

Note:
$$T_{finale}V_{final}^{k-1} = T_1V_2^{k-1} \Rightarrow T_{finale} = T_1 \left(\frac{V_2}{V_{final}}\right)^{k-1} = T_1 \left(\frac{V_1\tau^{-1}}{V_1\tau^{\frac{1}{k}-1}}\right)^{k-1} = T_1 \left(\tau^{\frac{-1}{k}}\right)^{k-1} = T_1\tau^{\frac{1-k}{k}}$$

N.A.: $T_{final} = 132.7 \text{ K} \text{ and also } -140.3^{\circ}\text{C}$

Energetic balance with the following hypotheses:

- The evolutions are adiabatic and reversible : $k = 1.4 = \gamma$
- There is not mechanical loss while storing or unstoring the energy (no frictions in the pressurized tank)
- No mechanical rubbings, nor in the transmissions between the wheels of the vehicle and the compressing device, nor in the compressor, nor in the pipes.
- The tank gets a volume of $V_1 = 20 L$.





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Commentaries:

• One immediately notes that the level of stored energy is very little regarding the wished 200 000 *J*, except with a tank of 400L, what is not reasonably usable in a car. However, the array below below gives the storable kinetic energy* and the corresponding speed of the vehicle (speed beyond which the storage of the kinetic energy will only be partial) according to the volume of the tank.

Volume of the tank (L)	20	40	50	60	100	150	200
Storable energy (J)	10157	20314	25393	30471	50786	76179	101571
Relative speed* (km/h)							
$v_{km/h} = 3.6 \times \sqrt{\frac{2\left(10157 \frac{V_{Liter}^{tank}}{20}\right)}{m_{kg}^{vehicle}}}$	16.2	23	25,6	28,1	36.3	44,4	51.3

*by neglecting the rubbings

For information : m = 1000 kg. With 60 L, acceptable volume, one could throw the vehicle until 28 km/h, what is *widely sufficient* in the cars' congestions.

- Moreover, this energy can be stored a priori during a short time.
- The output storage/unstorage is 100% if the re-launch of the vehicle is immediate after its stop. It is only 27% if the cooling of air is complete until the ambient temperature. Perhaps it will be necessary to work with a thermal insulated tank, what is not very penalizing in weight, but may be very cumbersome ... Moreover in this case, one sees that a very low temperature appears at the end of relaxation. It can be useful for the cooling of the car, for example by liquefying a frigorigen gas. Thus the compressor of the car's cooling system will work less and decrease the consumption of the car.

II.3.b) Utilisation d'une énergie hydraulique

• Choice of the fluid:

It will be a liquid fluid which will grant the energetic exchanges. The cheapest is water, but it can entail corroding and caviting effects inside the pipes. From these aspects, an oil is steadier, but it is more expensive too. However, if oil is circulating in a closed loop, the cost will not be a major problem.

Advantages:

- The hydraulic pumps or motors have a good output nearly 95%.
- There are reversible hydraulic motors/pumps :
 - While slowing down, they will be used as compressor of oil (kinetic energy \rightarrow hydraulic energy),
 - One will make them work as hydraulic motor to give back this energy while throwing again the vehicle (hydraulic energy \rightarrow kinetic energy). These technical aspects are currently studied on heavy trucks to reduce their fuel consumption.
 - The advantage of a reversible device is clear for compactness, and in addition, no synchronization will be required between 2 hydraulic dedicated devices.

The use of oil under pressure in the motor is very likely: while maintaining an important pressure in the tank, the hydraulic motor could be therefore a source of pressurized oil which could be common to specific functions of the thermal engines and/or of the vehicle:

- Lubrication of the thermal motor
- Regulation of the rates of compression inside the chambers of combustion
- Hydraulic accessories (assisted direction, hydropneumatic suspensions)
- To put gaz under pressure for another pneumatic device. One can *regroup besides* in a same vat under pressure oil it and gas while separating them by a flexible or mobile partition compartmentalizing the vat in 2 distinct tanks.

Drawbacks

- A vat of pressurized oil is quite dangerous in case of fire.
- Oil is almost incompressible and therefore cannot be pressurized by one of the rooms of combustion of the thermal engine :
 - To stop the pumping, one closes the 2 valves (intake and exhaust) permanently: in the case of gas, the motor can continue to run because gas can relax or compress.
 - If it is oil, *the motor will jam* because the volume of oil initially contained in the room while closing of the valves won't nearly vary.
 - This incompressible character of oil will impose to use a gaseous fluid or a spring to store the energy as the following diagram of working suggests it.

On the following page, the figure IV gives the general principle of circulation of pressurized fluids with recovery of energy within a vehicle :

- Many devices are optional; the <u>basic components are a reversible hydraulic motor</u>, an <u>auxiliary pump and a cylindrical vat containing air and oil under pressure.</u>
- The diagram shows a situation of energy storage: the vehicle decelerates and the wheels actuate the hydraulic motor which plays the role of *compressor for the air of the compartment 2*. If one uses *a hydraulic motor with reclining trays*, it is very easy to adjust the debit of oil entering in the compartment 1, or even to annul it, what will *prevent* the storage *so necessary*.
- As soon as the driver wishes to throw back the vehicle, the compressed air relaxes and hunt oil toward the hydraulic motor which then sends a mechanical energy toward the wheels, energy that will evidently not to be provided by the thermal engine as it is currently the case; thus an economy of fuel. Here also, the slant of the tray allows to pilot the quantity of oil leaving toward the motor, (which rejects oil to low pressure in the reservoir) and therefore to pilot the speed of unstorage of the energy contained in the compartment 2..

Permanently, the "accessories" and the lubrication extract oil in the compartment 1. A simple and inexpensive actionable auxiliary hydraulic pump (with gearings for example) will be necessary therefore to maintain the quantity and/or the pressure of oil necessary for the good working of the accessories and the lubrication. The working of this pump will be intermittent because the accessories should not use a lot of oil in relation to the quantities of oil pumped at the time of a deceleration. The small pumps may be actuated at will by the thermal motor or by an electric motor.

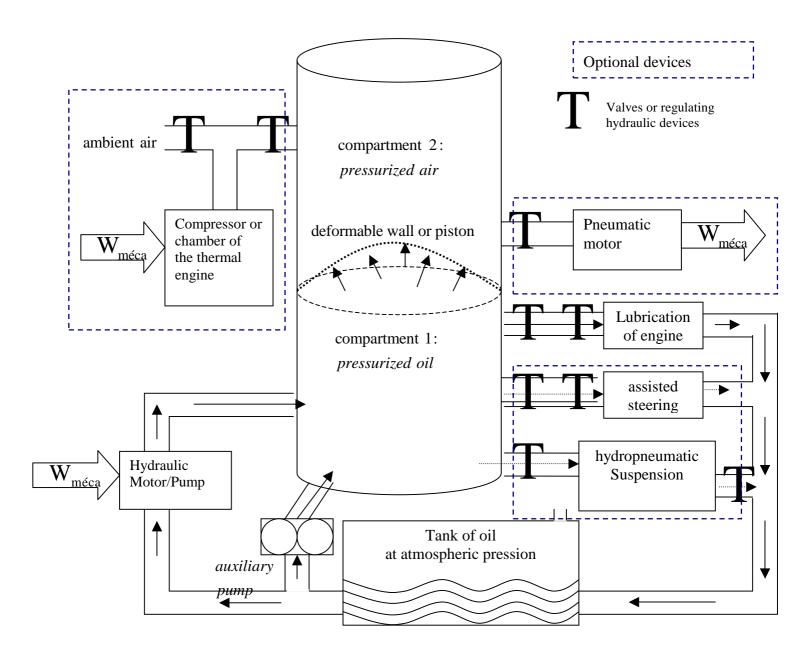


Fig. IV: General circulation of fluids with hydraulic recovery of energy inside the vehicle

II.4. The chemical storage

II.4.a) Short theorical descriptione

In a general way, an accumulator, as named abusively "reloadable battery" puts together 2 couples redox that reacts the one on the other according to a *reaction of oxydoreduction*:

$$\begin{array}{cccc}
ox_2 + n_2 & e^{-} & \stackrel{\rightarrow reduction \rightarrow}{\longleftarrow} & red_2 & (2) \\
red_1 & \stackrel{\rightarrow oxydation \rightarrow}{\longleftarrow} & ox_1 + n_1 & e^{-} & (1)
\end{array}$$

$$\Rightarrow n_1 ox_2 + n_2 red_1 \longleftrightarrow n_1 red_2 + n_2 ox_1 \\
(1) & \Rightarrow n_1 ox_2 + n_2 red_1 \longleftrightarrow n_1 red_2 + n_2 ox_1 \\
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The guidance in an electric circuit of the electrons resulting from the electronic exchanges during this reaction permits to get an electric current. The electromotive strength of this battery, as named "voltage" is estimated with the help of the formula $E = E_2 - E_1$: while supposing $E_2 > E_1$, E_2 is the electric potential of the cathode where take place the reductions and E_1 the one of the anode where take place the oxydations. The formulas of E_2 and E_1 are given by the Nernst's relation with "ai" the chemical activity of the species i and E°_j the standard potential of the couple redox n°_j :

$$E_{2} = E_{2}^{0} + \frac{RT}{n_{2}F} \ln \left(\frac{a_{ox2}}{a_{red2}} \right) \qquad E_{1} = E_{1}^{0} + \frac{RT}{n_{1}F} \ln \left(\frac{a_{ox1}}{a_{red1}} \right)$$

With F the Faraday's constant (96500 C/mol) , R the one of the perfect gases (8,32 J/K/mol) and T the temperature in Kelvin where the accumulator is working.

<u>Calculation</u>: the studied redox reaction is $n_1ox_2 + n_2 red_1 \longrightarrow n_1red_2 + n_2ox_1$. One has:

$$\begin{split} E &= E_2 - E_1 = E_2^0 + \frac{RT}{n_2 F} \ln \left(\frac{a_{ox2}}{a_{red2}} \right) - E_1^0 - \frac{RT}{n_1 F} \ln \left(\frac{a_{ox1}}{a_{red1}} \right) \\ E &= E_2^0 - E_1^0 + \frac{RT}{F} \left(\frac{1}{n_2} \ln \left(\frac{(a_{ox2})}{(a_{red2})} \right) - \frac{1}{n_1} \ln \left(\frac{(a_{ox1})}{(a_{red1})} \right) \right) \end{split}$$

This expression is not very used because one prefers to work with the reactional quotient Q:

$$E = E_{2}^{0} - E_{1}^{0} + \frac{RT}{n_{1}n_{2}F} \left(\ln \left(\frac{\left(a_{ox2}\right)^{n_{1}}}{\left(a_{red2}\right)^{n_{1}}} \right) - \ln \left(\frac{\left(a_{ox1}\right)^{n_{2}}}{\left(a_{red1}\right)^{n_{2}}} \right) \right)$$

$$E = E_{2}^{0} - E_{1}^{0} + \frac{RT}{n_{1}n_{2}F} \left(\ln \left(\frac{\left(a_{ox2}\right)^{n_{1}} \left(a_{red1}\right)^{n_{2}}}{\left(a_{red2}\right)^{n_{1}} \left(a_{ox1}\right)^{n_{2}}} \right) \right)$$
While putting
$$\frac{1}{Q} = \frac{\left(a_{ox2}\right)^{n_{1}} \left(a_{red1}\right)^{n_{2}}}{\left(a_{red2}\right)^{n_{1}} \left(a_{ox1}\right)^{n_{2}}} \text{ , we conclude } E = E_{2}^{0} - E_{1}^{0} - \frac{RT}{n_{1}n_{2}F} \ln \left(Q\right)$$

The accumulator will produce until the equating of the potentials of the electrodes $(E_2 = E_1 \Rightarrow E = E_2 - E_1 = 0)$; it can be seen also as the quotient of reaction equal to the constant K of thermodynamic balance: this balancing constant Qequilibre = K is available by knowing that $0 = E_2^0 - E_1^0 - \frac{RT}{n_1 n_2 F} \ln \left(Q_{equilibre}\right)$

In addition, we use $E^{\circ} = E^{\circ}_2 - E^{\circ}_1$ the standard electric strentgh of the accumulator.

$$0 = E_2^0 - E_1^0 - \frac{RT}{n_1 n_2 F} \ln \left(Q_{equilibre} \right) \implies \ln \left(K \right) = \ln \left(Q_{equilibre} \right) = n_1 n_2 \frac{FE^{\circ}}{RT}$$

II.4.b) Electric work of an accumulator

One studies again the reaction $n_1ox_2 + n_2 red_1 + n_1n_2e^- \longleftrightarrow n_1red_2 + n_2ox_1 + n_1n_2e^-$

One notes ξ the advancement of this reaction, defined with time t and number of moles of the species as it follows:

$$\xi\left(t\right) = \frac{\left(ox_{2}\right)_{t=0} - \left(ox_{2}\right)_{t}}{n_{1}} = \frac{\left(red_{1}\right)_{t=0} - \left(red_{1}\right)_{t}}{n_{2}} = \frac{\left(red_{2}\right)_{t} - \left(red_{2}\right)_{t=0}}{n_{1}} = \frac{\left(ox_{1}\right)_{t} - \left(ox_{1}\right)_{t=0}}{n_{2}}$$

During dt, one gets an elementary variation $d\xi$ of the advancement of reaction and $n_1n_2d\xi$ moles of electrons are exchanged. Then the accumulator provides:

- A voltage
$$E = E_2^0 - E_1^0 + \frac{RT}{n_1 n_2 F} \ln \left(\frac{\left(a_{ox2}\right)^{n_1} \left(a_{red1}\right)^{n_2}}{\left(a_{red2}\right)^{n_1} \left(a_{ox1}\right)^{n_2}} \right)$$

- A current
$$i = \frac{dq}{dt} = n_1 n_2 F \frac{d\xi}{dt} \Rightarrow idt = n_1 n_2 F d\xi$$
 avec $F = 96500$ C/mol.

The electric work which is algebraically got by the electric circuit plugged on the accumulator is then:

$$\delta W_{elec} = Eidt = \left[E_2^0 - E_1^0 + \frac{RT}{n_1 n_2 F} \ln \left(\frac{\left(a_{ox2}\right)^{n_1} \left(a_{red1}\right)^{n_2}}{\left(a_{red2}\right)^{n_1} \left(a_{ox1}\right)^{n_2}} \right) \right] \left[n_1 n_2 F d\xi \right]$$

The reaction will stop when electromotrive strength of the accumulator becomes 0. As the activities of the chemical species depend *a priori* of the advancement of reaction, the advancement until an infinite time, $\xi(t=\infty)$, will be computed with the help of the following formula, while supposing that the disappearance of one of the reactive species will not jam the reaction before it has reached its thermodynamic balance:

$$\frac{n_{1}n_{2}F\left(E_{2}^{0}-E_{1}^{0}\right)}{RT} = \ln\left(\frac{\left(a_{ox2}\left(\xi_{\infty}\right)\right)^{n_{1}}\left(a_{red1}\left(\xi_{\infty}\right)\right)^{n_{2}}}{\left(a_{red2}\left(\xi_{\infty}\right)\right)^{n_{1}}\left(a_{ox1}\left(\xi_{\infty}\right)\right)^{n_{2}}}\right)$$

One experiments in thermodynamics that:

- The activity of a gas is roughly its partial pressure (Pa) on $P^{\circ}=10^{5}$ Pa
- The activity of a species in solution is roughly its concentration in mol/L
- The activity of a solid is roughly 1.

And the total electric work of that the accumulator will provide is:

$$W_{\text{elec}} = \int_{0}^{\xi_{\text{so}}} \delta W_{\text{elec}} = \int_{0}^{\xi_{\text{so}}} \left[E_{2}^{0} - E_{1}^{0} + \frac{RT}{n_{1}n_{2}F} \ln \left(\frac{\left(a_{ox2}(\xi)\right)^{n_{1}} \left(a_{red1}(\xi)\right)^{n_{2}}}{\left(a_{red2}(\xi)\right)^{n_{1}} \left(a_{ox1}(\xi)\right)^{n_{2}}} \right) \right] \left[n_{1}n_{2}Fd\xi \right]$$

Let note here that, W_{elec} , work given to the circuit, is algebraic:

- If $n_1ox_2 + n_2 \ red_1 + n_1n_2e^- \rightarrow n_1red_2 + n_2ox_1 + n_1n_2e^-$, reaction is in the direct and spontaneous sense $\xi_{\infty}>0$. W_{élec} >0. There is *consumption of reactive species* in the accumulator which *unloads itself*.
- If $n_1ox_2 + n_2 red_1 + n_1n_2e^- \leftarrow n_1red_2 + n_2ox_1 + n_1n_2e^-$, reaction is in the indirect and forced sense $\xi_{\infty} < 0$. $W_{\text{elec}} < 0$, There is regeneration of reactive species in the accumulator which loads itself. This is electrolysis where the circuit provides electric energy to the accumulator.



II.4.c) Example: the plomb-accumulator

It is used by the battery for automotive applications. The chemical principle of this type of accumulator is a redox reaction between the 2 following couples: Pb^{2+}/Pb , PbO_2/Pb^{2+} where the Pb has oxydation's number of respectively +II, O, +IV,+II.

When the engine doesn't work and the battery is solicited (while starting engine, lights or radio...), it works as an accumulator which unloads:

- the + connection is the cathode where PbO₂ is reduced $PbO_{2s} + 4H^+ + 2e^- \rightarrow Pb_{aq}^{2+} + 2H_2O$
- the connection is the anode where Pb is oxydized : $Pb_s \rightarrow Pb_{aq}^{2+} + 2e^{-}$

Balance:
$$Pb + PbO_{2s} + 4H^+ \rightarrow 2Pb_{aq}^{2+} + 2H_2O$$

Whereas if the engine is running, it actuates an alternator which regenerates the battery while loading it.

- the connection is the cathode where Pb²⁺ is reduced $Pb_{aq}^{2+} + 2e^- \rightarrow Pb_s$
- the + connection is anode where Pb²⁺ is oxydized : $Pb_{aq}^{2+} + 2H_2O \rightarrow PbO_{2s} + 4H^+ + 2e^ Balance : Pb_s + PbO_{2s} + 4H^+ \leftarrow 2Pb_{aq}^{2+} + 2H_2O$

One will note [A] the concentration in mol/L of the chemical species A.

An electrolytic cell with plomb usually uses sulfuric acid $(2H^+SO_4^{2-})$ as electrolyte with a concentration or roughly $c_0 = 1,7$ mol/L. One notes V_{cell} the volume of electrolyte contained in the cell. The big quantity of SO_4^{2-} always grants the presence of solid plomb's sulphate in the solution; thus one has $\lceil Pb^{2+} \rceil \lceil SO_4^{2-} \rceil = K_S$ which is the solubility product of $PbSO_{4s}$.

The potential at the cathode is
$$E_2 = E_2^0 + \frac{RT}{2F} \ln \left(\frac{\left[H^+\right]^4}{\left[Pb^{2+}\right]} \right) = E_2^0 + \frac{RT}{2F} \ln \left(\frac{\left[H^+\right]^4 \left[SO_4^{2-}\right]}{K_s} \right)$$

The potential at the anode is
$$E_1 = E_1^0 - \frac{RT}{2F} \ln \left(\left[Pb^{2+} \right] \right) = E_1^0 - \frac{RT}{2F} \ln \left(\frac{K_s}{\left[SO_4^{2-} \right]} \right)$$

Thus it gives the voltage:
$$E = E_2 - E_1 = E_2^0 - E_1^0 + \frac{RT}{2F} \ln \left(\frac{\left[H^+\right]^4 \left[SO_4^{2-}\right]}{K_S} \cdot \frac{K_S}{\left[SO_4^{2-}\right]} \right) = E^0 + \frac{2RT}{F} \ln \left(\left[H^+\right] \right)$$

With $E_2^0 - E_1^0 = E^\circ$ about 2.1 V; therefore, a 12V battery has 6 cells linked in series. Here, the concentration of H⁺ varies according to the advancement $\xi: [H^+] = c_0 - 4\frac{\xi}{V}$ in mol/L

Thus the electric work
$$W_{elec} = \int_0^{\xi_{\infty}} \left[E^0 + \frac{2RT}{F} \ln \left(\left[H^+ \right] \right) \right] 2F d\xi$$

The top advancement is reached when E = 0. If it works at a temperature of 25°C, either T = 298K:

$$E^{0} + \frac{2RT}{F} \ln \left(\left[H^{+} \right] \right) = 0 \Rightarrow \left[H^{+} \right]_{t=\infty} = \exp \left(-\frac{FE^{0}}{2RT} \right) = 1,784.10^{-18} mol$$

Practically all the acid reacted, therefore the final advancement is $\xi_{\infty} = V_{cell} \frac{c_0}{4}$ in mol.

Thus:
$$W_{\text{\'elec}} = \int_0^{\xi_{\infty}} \left[E^0 + \frac{2RT}{F} \ln\left(\left[H^+\right]\right) \right] 2F d\xi$$
, soit $W_{\text{\'elec}} = V_{\text{cell}} \left[\frac{FE^0 c_0}{2} + RT \left(c_0 \ln c_0 - c_0\right) \right]$

By choosing a volume of 100 mL per cell for acid and an initial concentration of 1,7mol/L, it works: $W_{elec}^{1cell} = 17~027, 4~J$ for each cell, and for a 6 cells battery: $W_{elec}^{1bat} = 102~164, 5~J$

Such a battery can produce *a priori* a quantity of charges: $Q = 6\frac{V_{cell} c_0}{4} 2F$ because 2F Coulomb are exchanged for each mole of advancement and the final advancement is in mole $V_{cell} c_0/4$ in each cell. Either Q = 49 215 C. If one wishes to have it in « ampere.hour », knowing that 1 A.h = 3600 C, for such a battery, "top load" provides an autonomy of 13.67 A.h.

The volume of this battery is about $15 \times 13 \times 10 = 1950 \text{ cm}^3$. As the hypothesis of 20 L dedicated to the storage, it is sufficient for 10 batteries, either an energy of $10^6 \text{ J} \cdot 19 = 10^6 \text{ J} \cdot 19 = 10^$

$$\frac{1}{2}mv^{2} = 10 \ W_{elec}^{1bat} \Rightarrow v = \sqrt{\frac{2 \times \left(10 \ W_{elec}^{1bat}\right)}{m}} = 45,2 \ m/s = 162,73 \ km/h$$

On the other hand, <u>batteries can be slow to load or unload</u>: the required time to load depends on its design because of a maximal current of electrolysis. Let's say here that these loads are demanding *a source of direct current*: either a continous electric motor used as a generator, either an alternative generator while straightening with sometimes cumbersome devices, what isn't the ideal on a car ... The current of electrolysis depends on complex laws and:

- the area of the electrodes in contact with the electrolytique liquid
- the material of the electrodes and its surfacic features
- the voltage of electrolysis put down to the connections of battery.

The current can be high if the surface of electrodes dove in the electrolytic liquid is big. For example for the metallurgy of zinc by electrolytic way, one can go until 4 A/dm2 of electrode for the electrolysis of a solution of zinc sulphate $(z_n^{2+}SO_4^{2-})$ with an anode in argentiferous plomb and a cathode made of aluminum under a tension of 3,4 V. But the voltages of electrolysis on a cell must be weak (in general < 5V). An alternative to this problem is to put in series the cells, what divides the "global" voltage of electrolysis by the number of cells in series.

With regard to the time of discharge, it can be very variable according to the solicitation of the battery. However, an "incompressible" discharge time exists because of the existence of a maximum current of discharge: this current is influenced by the same parameters which have on it appreciably the same effects. Thus, for a battery of $15 \times 13 \times 10$ cm:

- by using for each electrode 3 equidistant sheets of 12 x 10 cm : $S_{electrode} = 720 \text{ cm}^2$ because each sheet has 2 sides of 120 cm².
- By supposing $I_{surfacique} = 4 \text{ A/dm}^2$

We get a current of electrolysis of 28,8 A. Thus it will need at least 13,7/28,8 = 0,47 h to reload completely a une battery, nearly ½ hour! With 10 batteries *in parallel*, the currents add themselves and one can rise to 288 A of total current of electrolysis. The stored energy by only one battery during 30 min is thus distributing on the 10 *but in 3 min*. What is again long as a slowing down time in a car with a current probably not reasonable...



Let's think that some car makers as Honda bent on the difficulties of energy recovery by electrolytic storage on their battery on their vehicles with fuel cell, notably the Honda FCX (4th generation). Owing disappointing energizing outputs in deceleration, Honda developed a "supercaps" of a 8 Farad capacity which stocks the current produces by the electric continuous motor of the vehicle (which works like a generator while slowing down). This electric energy is stored under the shape of an electrostatic energy in the elements of the" supercaps ". It would seem that the output is very good (> 95%), but it subsists a problem of clutter and especially of costs. On the technical plan, a supercaps of 34 kg can deliver during some seconds a power of 40 kW to throw back the vehicle, either about 55 Hp. But its volume occupies the half of the case of the car. However, the concept stays interesting to throw back the vehicle between 2 red lights. Regarding that, it is a serious alternative to the electrolytic storage: electrolytic storage is slow, and in addition entails polluting problems because the batteries are using a lot of agressive chemical matters (acidic, various heavy metals), contrary to the capacitors.

III. Ordering of the storage technologies

III.1. Development of criterias to order

One tries here to think about the relevance of a storage system in relation to another. The *ideal* storage system should have the following qualities :

- High energetic output:
 - o 100% of the initial kinetic energy has to be stored,
 - o 100% of this energy has to be given back.
- Fast storage and unstorage of the energy
 - The kinetic energy must be strored in the same time as a classical braking with mechanical rubbing brakes.
 - The stored energy must be released on the same time as the acceleration given by the thermal engine of the vehicle,
- Easiness to transmit the power towards the wheels:
 - o The system should not need for kinematic complex elements (gearings, clutches, CVT ...) to give back the stored energy towards the wheels.
- High maximal energy stored
- Reduced clutter
- Little weight
 - o The weight is bas for vehicles without recovering their kinetic energy:
 - Oversizing of the suspensions
 - Wear of the tires
 - Huge energetic losses at every braking
 - The weight is quite less penalizing if the kinetic energy of the vehicle is recovered while braking.

Linked with the vehicles, some of these criterias are less important than of others: in a train or a truck, the weight and the clutter of the system are less bothersome than on a small car. Inversely, the speed of storage / unstorage is primordial on a car (numerous short accelerations / decelerations) whereas on a train, it is less necessary (long accelerations or decelerations, and often few). In the same way, the maximal energy to store depends directly on the mass and the speeds of cruise of the vehicles (very big for a TGV, weak for a city-car). And nevertheless, energetic output has to be improved whatever the vehicle.



Thus it is difficult to to classify these criterias by order of importance in an absolute way. Nevertheless, one can keep the "typical" ordering next one in the case of small vehicles whose speed very often varies during a journey:

- 1. High energetic output
- 2. Fast storage/unstorage of the energy
- 3. Easiness to send the energy towards the wheels
- 4. Reduced clutter
- 5. High maximal stored energy
- 6. Small weight

III.2. Notation of every solution

III.2.a) Principle

For every criteria, one mark from 1 to 5 has been allocated for each solution

5 : very good

4 : good

3 : middle

2 : bad

1 : very bad

Moreover, the following coefficients are given to each criteria relatively to their importance:

Criteria n°1 : coeff. 6
Criteria n°2 : coeff. 5
Criteria n°3 : coeff. 4
Criteria n°3 : coeff. 4
Criteria n°6 : coeff. 2

Maximal brut mark : $23 \times 5 = 115$. Final mark based on $20 : N = Brut mark \times 20 / 115$

III.2.b) Notation of the wheel with high inertia

Criteria n°	Coeff	Mark	Points	Commentary
Energetic output	6	4	24	The mechanisms to transmit the power may decrease the good
				behaviour of the inertia wheel in itself
Fast storage and	5	3	15	The synchronisation between the velocities of inertial wheel and
unstorage				rolling wheels will not be instantaneous
Easiness to recover				
the energy	4	1	4	It requires clutch or continuously variable transmission.
Reduced clutter	4	2	8	The clutter of transmissions attenuates the compactness of the
				inertia wheel.
Maximal storable	2	5	10	The best approach of storage according to this criteria.
energy				
Weight	2	1	2	The mass of the transmission adds to the high mass of the inertia
				wheel.
				A choice which has big problems of weight and mechanical
Mark on 20	10,95	Total	63	complexity, but may be useful for heavy vehicles whose speed is
				not oscillating a lot.

III.2.c) Notation of the spiral spring

Criteria n°	Coeff	Mark	Points	Commentary
Energetic output	6	5	30	The compression or the relaxation of a spring has an output of
				roughly 100%.
Fast storage and	5	5	25	The fast relaxation or compression of the spring are not difficult a
unstorage				priori.
Easiness to recover				The relaxation of the spring makes rotate the wheels in the wrong
the energy	4	3	12	sense while unstoring. But wheels have to turn in the same way
				while storing and unstoring An intermittent device is necessary
				to reverse the motion.
Reduced clutter	4	3	12	The clutter of the spiral spring is not big while estimating the
				stored (Cf. II.2.b).
Maximal storable	2	3	6	Correct energy correcte, but 4 times smaller than the inertia
energy				wheel for a same volume of storage.
Weight	2	4	8	The mass of the spring may be weak, but the one of the
				transmission is difficult to value here.
				An attractive solution as regards to its output and its speed of
Mark on 20	16,17	Total	93	storage / unstorage. But shading zones subsist on the resistance of
				the spiral to turn several rounds and on the system of restitution
				of the energy toward the wheels.

III.2.d) Notation of the traction spring

Criteria n°	Coeff	Mark	Points	Commentary
Energetic output	6	5	30	The compression or the relaxation of a spring has an output of
				roughly 100%.
Fast storage and	5	5	25	The fast relaxation or compression of the spring are not difficult a
unstorage				priori.
Easiness to recover				The relaxation is a sliding motion with limited stroke (some cm):
the energy	4	2	8	it is not optimal to rotate continously a rolling wheel.
				The clutter of the helical spring is not big while estimating the
Reduced clutter	4	2	8	stored (Cf. II.2.b), but less good than the spiral spring.
Maximal storable	2	1	2	Energy too much weak: 13 times less the one of the inertial
energy				wheel for a same volume of storage.
Weight	2	4	8	The mass of the spring may be weak, but the one of the
				transmission is difficult to value here.
				An attractive solution as regards to its output and its speed of
Mark on 20	14,08	Total	81	storage / unstorage. But a weak stored energy because the central
				zone of the spring is lost and the probable need for materials with
				very high elastic limit.

III.2.e) Notation of the pneumatic/hydraulic storage

Criteria n°	Coeff	Mark	Points	Commentary
Energetic output	6	4	24	The compression or relaxation of a gas give an output of roughly
				100%, but under the condition to prevent the cooling of gas. The
				hydraulic motors have an excellent output (>95%).
Fast storage and	5	5	25	The fast relaxation or compression of the air are not difficult a
unstorage				priori.
Easiness to recover				The relaxation of the gas sends the oil inside the hydraulic motor
the energy	4	4	16	which converts easily this pressure into a continous rotary motion
				with high torque/low speed, moreover a <i>variable</i> speed at will.
				The clutter is considerable while regarding the stored energy. But
Reduced clutter	4	2	8	the oil tank can integrate numerous functions useful for the
				vehicle, what attenuates this drawback.
Maximal storable	2	1	2	Energie faible en particulier si l'air se refroidit avant de se
energy				détendre.
Weight	2	4	8	The weight may be quite small because few transmitting elements
				are needed.
				An attractive solution as regards to its output and its speed to
Mark on 20	14,43	Total	83	store/unstore. In spite of storing a lot of energy, it is the approach
	, -			which gives the most of easiness to unstore it.

III.2.f) Notation of the chemical storage

Criteria n°	Coeff	Mark	Points	Commentary
Energetic output	6	4	24	The output of electrolysis is often good and the discharge of batteries too (>80%)
Fast storage and unstorage	5	3	15	If the unload of a battery can be quick, the time to load is too long while taking the time of braking, for example in a car.
Easiness to recover the energy	4	5	20	The unload of an accumulator makes often itself easily, and even quickly. The generator, which firstly has load the accus, becomes secondly an electric motor with direct current. In addition, this motors give high torque at low RPM, what is the ideal to throw a vehicle (complex transmission are useless between the motor and the rolling wheels).
Reduced clutter	4	5	20	The clutter of batteries is relatively weak and the rotary motion given by an electric motor doesn't need cumbersome transmissions.
Maximal storable energy	2	5	10	The best way according to this criteria.
Weight	2	3	6	The weight of the accumulators, event recent, stays still important (10 kg for a car) and one should use 10 of them.
Mark on 20	16,52	Total	95	An attractive solution as regards to the storable energy and its easiness of recuperation. However, the weight remained penalizing and the times of reload are too much long.

III.3. Summary

III.3.a) Ordering of the technologies and notes

One finally gets for usual cars the following ordering:

- 1. The chemical storage : 16,52/20 2. The storage with spiral spring : 16,17/20
- 3. The pneum/hydraulic storage: 14,43 4. The storage with helical spring: 14,08 /20
- 5. The storage with inertia wheel: 10,95/20

As we have already noticed it, these solutions give very different advantages and drawbacks, what doesn't appear in the final mark. Rather than to oppose these solutions between them, it is worth to better wonder about the possibility to join the effects of at least 2 among them on a same vehicle. Indeed, it seems very applicable to use 2 systems:

- The one, slow but with strong ability of storage, efficient for long slow down or prolongated deceleration,
- The other, efficient for short decelerations (few energy to store, but quickly), for example in urban automotive environment.

In addition, some of these systems can become only one: for example, compressed air can be replaced by compressing a spring, and better, one can compress a spring in a compartment containing air *too*, in order to get *both* elastic *and* pneumatic energies *in a same volume*, approximately cylindrical.

At last, some of these systems bring fonctionalities which *have to be done by other devices* whatever the vehicle: this is the case, on the one hand, of the electric accumulators with chemical storage, because electrical energy is indispensable in a car (sparkling, lights, radio, onboard computer ...), and on the other hand, hydraulic energy is often very useful (lubrificating, assisted steering or suspension...). Thus, even if a device looks like cumbersome in itself, it could free space somewhere, what will attenuate this negative aspect.

Now regarding the technological know-how of these solutions, the chemical storage rises a lot since many years with regular breaktrough (lithium-ions, lithium-metal-polymers technologies ...); it entails the recent development of hybrid reloadable cars and of the portable electronics (computer, cellphones, MP3...). The storage with inertia wheel is known since a lot of years, but hasn't overcome to rise. At last, it seems that storage with mechanical springs hasn't concrete realization, except the little toy-cars for children.

III.3.b) What system to recover kinectic energy on a car?

As we have already seen it, inertia wheels are foreseeable only on sufficiently big and heavy vehicles with high kinetic energy (trucks or trains). It doesn't seem to be reasonable to use it for cars. Moreover, to use 2 storing systems looks like a good choice because there is no system with high level of storage which were fast to store/unstore the energy. Another fact is that chemical storage is useful and could lead to the 42V architecture. This architecture has been elected at a time for the energizing gains that it offers (by reduction of the Joule effect) but is not more currently on the agenda. Finally, the hydraulic storage also brings numerous and often necessary functionalities for the vehicle.

In SYCOMOREEN's opinion, the most suitable system for a car seems to be composed therefore:

- Of a fast pneumatic/hydraulic storage with pistons (the piston could possibly pull/tow a spring) or a device witth spiral spring: the pistons can be those of the MPRBC or POGDC engines with dedicated chamber and a tank of compressed air.
- Of a slow but powerful storage, with electric accumulators.

Conclusion

This annex allowed to draw the main lines of what could be the devices of energetic storage for hybrid vehicles of which one of the way of propulsion is thermal. However, it is well obvious that this first approach would likely to be worked again with a more rigorous survey of the different systems of proposed storage and an exhaustive research of the foreseeable means of storage.

Let's say again that the first goal of *SYCOMOREEN* in the domain of motors/pumps is to design engine which are compact enough so as *the implantation of such devices* in a vehicle were not *utopian anymore*, but on the contrary, *seriously foreseeable*; on the one hand, *the counter-argument of the clutter* of hybrid systems *isn't not more receivable* while seeing the large fuel economy which is brought by the recovery of kinetic energy within the vehicle thanks to the high compactness of the thermal engine, and on the other hand, we've noticed that with 20 000 cm³ (volume of a small suitcase) dedicated to the storage (not included the possible mechanical transmission), we succeed to store an energy able to launch a vehicle of 1000 kg between 16 and 160 km/h, with an « average » of 35 km/h, what is clearly sufficient in an automotive urban situation.

The *most difficult step*, which is the design of hyper-compact architectures of thermal engines, *has been cleared*, and with no doubt, the fabrication and the improvement of the MPRBC and POGDC engines will open *numerous ways of thinking and research* able to *deeply change the working and the perception of vehicles for the 3rd millennium*.

